ON THE FLUCTUATIONS OF THE SURFACE BRIGHTNESS OF GALAXIES*

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Abstract. We consider the problem of correlation between the surface brightnesses of two nearby points for an external galaxy under certain simplifying assumptions. A probability density function for these intensities are derived, as are its moments and correlation coefficient.

1. Introduction

About fifty years have elapsed since the time when the idea was put forward that the absorbing matter in our Galaxy consists of a large number of discrete absorbing clouds of different optical thickness (see Ambartsumian and Gordeladze, 1938). The clouds of large optical thickness we often observe as the 'dark nebulae' or 'dark spots' in the Milky Way, The clouds of the optical thickness $\tau < 0.5$ are causing local decreases in the number of stars of certain apparent magnitudes per square min of arc. Thus clouds of both kind determine the differences of surface brightness of the Milky Way which conditionally bear the name of 'fluctuations of surface brightness'.

The construction of more or less realistic theory of such fluctuations encounters great difficulties. Therefore, some very simplified mathematical models have been studied. Thus the model was considered in which the stars are distributed in the infinite space with such uniformity that we can substitute them by a continuous luminous medium having a constant coefficient of emission η, which is independent of coordinates, while the absorbing clouds are distributed at random but uniformly in the sense that the probability of the presence of the centre of a cloud in a volume element is equal to this volume multiplied by a constant coefficient. In such a model the absorbing properties, the shapes and the sizes of clouds are to be described by different parameters and the statistical distribution of those parameters must be given. It is supposed that this distribution is independent of the coordinates of the centre of the cloud. For example we can have a set of spherical clouds of a given radius (or of different radii) that are distributed at random (according to Poisson law) in the space. However, the sizes of clouds have to be negligibly small as compared with the distances between them.

Under these assumptions an equation for the probability distribution function for the values of brightness at a point of Milky Way has been derived and the mean values of brightness and of the square deviation have been found (Ambartsumian, 1944).

Comparing the obtained formal expressions with the observations it was possible to

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estimate the approximate values of some parameters describing the statistics of absorbing clouds.

However, the problem of correlation between the surface brightnesses of two mutually nearby points of Milky Way, some finite angular distance α apart has not been solved. It appears that it requires some complicated calculations.

In this paper we consider the correlation problem for a somewhat different model, where we deal with an external spiral system (external galaxy), when the rays directed to the observer from two points of the system are almost parallel in so far as we remain within the system or in its neighbourhood. We shall assume also that the optical depth traversed by each of the rays before they reach the boundary directed to the observer is infinite. It follows that the results obtained from consideration of such a model are applicable mainly to the cases when the observer is situated near the equatorial plane of the external galaxy. This means that he sees the spiral system edge on. The examples of such systems are NGC 4594 (M104), NGC 4244, and NGC 4631.

Though in a system described above the volume filled by absorbing clouds and emitting matter (stars) is only a half space, the problem formulated is equivalent to the problem of correlation between intensities for two parallel rays (distance r apart) in the homogeneous medium of the same kind, but filling the entire space. But of course, we are interested only in the correlation between intensities in points of intersection of our two rays with a plane perpendicular to them.

2. The Equation for the Probability Distribution

Evidently we can apply here the same *invariance principle* which was used by us (see Ambartsumian, 1944) in the case of one single ray. Thus our problem is to find the probability density function $u(x_1, x_2; r)$ of the intensities x_1 and x_2, in the corresponding points of our two rays.

According to invariance principle when we displace both points by an amount ΔS in the same direction along the corresponding rays, the two-dimensional distribution function of the two quantities x_1 and x_2 must remain unchanged. This means that the sum of all possible changes equals zero. From this and passing to the limit $\Delta S \to 0$ we obtain – similarly as in the paper just quoted, the equation

$$u(x_1, x_2; r) + \frac{\eta}{\nu(r)}\left(\frac{\partial u}{\partial x_1} + \frac{\partial u}{\partial x_2}\right) = \int\int u\left(\frac{x_1}{q_1}, \frac{x_2}{q_2}; r\right) \frac{\mathrm{d}^2\phi_r(q_1, q_2)}{q_1 q_2}, \quad (1)$$

where $\nu(r)$ is defined by the condition that $\nu(r)\Delta S$ is the probability of intersection of at least one of two rays on the range ΔS by some absorbing cloud, while $\phi_r(q_1, q_2)$ is the conditional probability that in this case the intensity of the first ray will be multiplied by a transmission coefficient that is $\leqq q_1$ while the intensity of the second ray by a coefficient $\leqq q_2$. Thus we assume that generally the transmission coefficients of a cloud when passing it at different points are different.

We assume that the clouds may differ in their shapes and transparencies, but they form a homogeneous set, which can be described by one transparency function $\phi_r(q_1, q_2)$ as defined above. It is clear that, in our model,

$$\phi_r(q_1, q_2) = \phi_r(q_2, q_1) \tag{2}$$

describes the optical-statistical properties of the whole set of absorbing clouds.

For the following it is convenient to introduce the dimensionless intensities

$$y_1 = \frac{\nu x_1}{\eta}\,; \qquad y_2 = \frac{\nu x_2}{\eta}\,. \tag{3}$$

Then for their distribution function we shall have the equation

$$u + \frac{\partial u}{\partial y_1} + \frac{\partial u}{\partial y_2} = \int\!\!\int u\left(\frac{y_1}{q_1}, \frac{y_2}{q_2}; r\right) \frac{\mathrm{d}^2\phi_r(q_1, q_2)}{q_1 q_2}\,, \tag{4}$$

from which we can easily obtain the expressions for the expectations of products $y_1^k y_2^l$.

3. The Moments and the Correlation Coefficient

If we multiply (4) by $y_1^k y_2^l$ and integrate over all possible values of y_1 and y_2 we obtain the relations from which we can find the values of corresponding mathematical *expectations* (moments). The most interesting fact is that the moments $\overline{y_1^k y_2^l}$ derived in this way are expressed in simple ways through the different moments of products of the type $q_1^n q_2^m$. Owing the symmetry condition (2) we always have

$$\overline{y_1^k y_2^l} = \overline{y_1^l y_2^k}\,; \qquad \overline{q_1^m q_2^n} = \overline{q_1^n q_2^m}\,. \tag{5}$$

Since, in particular

$$\overline{y_1^k} = \overline{y_2^k}\,; \qquad \overline{q_1^m} = \overline{q_2^m}$$

it is convenient to write simply $\overline{y^k}$ instead of $\overline{y_1^k}$ and of $\overline{y_2^k}$ as well as $\overline{q^k}$ instead of $\overline{q_1^k}$ and $\overline{q_2^k}$.

As the result of calculation we obtain

$$\bar{y} = \frac{1}{1 - q}\,; \qquad \overline{y^2} = \frac{2}{(1 - \bar{q})(1 - \overline{q^2})}\,;$$
$$\overline{y_1 y_2} = \frac{2}{(1 - \bar{q})(1 - \overline{q_1 q_2})}\,. \tag{6}$$

Thus for the ratio of moments we have

$$\rho = \frac{\overline{y_1 y_2}}{\overline{y^2}} = \frac{1 - \overline{q^2}}{1 - \overline{q_1 q_2}}\,, \tag{7}$$

and for the correlation coefficient

$$K = \frac{\overline{(1 - q_1)(1 - q_2)}}{(1 - q)^2} \; \frac{1 - \overline{q^2}}{1 - \overline{q_1 q_2}} \; . \tag{8}$$

4. Example

Let us consider for simplicity an artificial example. We suppose that all clouds are thin square plates of equal size and of equal transparency coefficient q_0, when the rays are perpendicular to them. Let l is the side of the squares. The corresponding sides of squares are parallel each other and the plane of each square is perpendicular to the direction of the ray. One of the sides of squares is parallel to the line connecting the points of rays at which the intensities we consider. The centre of the squares are distributed in space at random, according to Poisson law.

If a given cloud intersects one of our rays the probability that it intersects also the second ray will be $(l - r)/(l + r)$, while the probability of intersecting of only one of the rays is $2r/(l + r)$. In this case we have

$$\overline{q_1} = \overline{q_2} = \frac{l}{l + r} \, q_0 \; ; \qquad \overline{q_1^2} = \overline{q_2^2} = \frac{l}{l + r} \, q_0^2 \; ; \qquad \overline{q_1 q_2} = \frac{l - r}{l + r} \, q_0^2 \tag{9}$$

and according (7) we obtain

$$\rho = \frac{l(1 - q_0^2) + r}{l(1 - q_0^2) + r(1 + q_0^2)} \; . \tag{10}$$

If for example $r = \frac{1}{2} l$, $q_0 = \frac{1}{2}$ the correlation coefficient, $K = \frac{25}{33}$. Thus the correlation in this case is sufficiently strong.

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